BBD-003-1164004 Seat No. ________

M. Sc. (Sem. I) Examination

July - 2021

Mathematics : CMT - 4004

(Graph Theory)

Faculty Code : 003

Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal marks.

1 Answer following seven questions : **7×2=14**

1. Define terms : Odd vertex, Even vertex, Regular graph, k–Regular graph.
2. Define terms : Graph, Degree of a vertex, Self loop, Parallel edges (Multiple edges).
3. Define terms : Tree, Acyclic graph, Pendent vertex, Pendent edge.
4. Give two non isomorphic graphs G_1 and G_2, which are having properties $|V(G_1)| = |V(G_2)|$, $|E(G_1)| = |E(G_2)|$ and for any non-negative integer t, the number of vertices in G_1 with t degree and the number of vertices in G_2 with t degree are same.
5. Give two example of Eulerian graphs G_1 and G_2, so that G_1 is Hamiltonian graph, while second graph G_2 does not admits any Hamiltonian cycle.
6. Define term: Arbitrary traceable from a vertex in an Euler graph.
7. Give an example of an Eulerian graph, which has two arbitrary traceable vertices.

2 Answer following seven questions : **7×2=14**

(i) Prove or disprove that, any tree T with $|V(T)| \geq 2$ has atleast two pendent edges.

(ii) Prove or disprove that, any tree T with $|V(T)| \geq 2$ has precisely two pendent vertices.

(iii) Define distance between two vertices in a connected graph.

(iv) Define term: Eccentricity of a vertex in a connected graph and center of a connected graph.

(v) Write down atleast three facts about dual of a planner graph.

(vi) Define term: Incidence matrix of a graph.

(vii) Write down atleast three properties of an incidence matrix of a graph G.

3 Answer following two questions : **2×7=14**

(a) Let G be a finite graph. Prove that there are subgraphs $g_i = (V_i, E_i)$, $i = 1, 2, \ldots\ldots, k$, for some $k \geq 1$ such that,

(i) Each g_i is a maximal connected subgraph of G.

(ii) $V_i \cap V_j = \phi$, $i \neq j$ and $i, j \in \{1, 2, \ldots, k\}$

(iii) $V = V_1 \cup \ldots \cup V_k$ and $E = E_1 \cup \ldots \cup E_k$.

(iv) If $g = (W, F)$ be any connected subgraph of G, then g must be a subgraph g_i, for some $i \in \{1, 2, \ldots, k\}$.

(b) Let $G = (V, E)$ be a graph. Prove that, G is a disconnected graph if and only if there are two disjoint subsets V_1 and V_2 of V such that, (i) $V = V_1 \cup V_2$ and (ii) there is no edge uv in G, whose one end vertex lies in V_1 and another end vertex lies in V_2.

4 Answer following one question : **1×14=14**

Let G be a connected graph. Prove that, it is an Euler graph if and only if its all the vertices have even degree.

5 Answer following two questions : **2×7=14**

1. Prove that, for a separable graph G, v is a cut vertex in G if and only if there are two vertices $x, y \in V(G) - \{v\}$ such that every path in G between x and y passes through v.

2. Let $X(G) = \begin{vmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{vmatrix}$

   be an adjacency matrix of a graph G. Find $Y = X + X^2 + X^3 + X^4$, where $X = X(G)$. Determine G is a connected graph or it is not a connected graph.

6 Answer following two questions : **2×7=14**

(a) Let G be a connected graph and S is a cut set for G. Let T be a spanning tree for G. Prove that, $S \cap E(T) \neq \phi$..

(b) Let G be a connected graph and S be a cut set for G. Let F be any cycle in G. Prove that, $|E(F) \cap S|$ is even.

7 Answer following two questions : **2×7=14**

1. Prove that, a graph G is a minimally connected graph if and only if it is a tree.
2. Prove that, a connected graph G, admits a spanning tree.

8 Answer following two questions : **2×7=14**

(a) For a tree T, with $|V(T)| = n$, prove that, T has $n - 1$ edges.

(b) Let G be a connected graph and it satisfy $|E(G)| = |V(G)| - 1$. Prove that, G is a tree.

9 Answer following one questions : **1×14=14**

Define term: Maximal non-Hamiltonian graph. Also State and prove Dirac's Theorem.

10 Answer following two questions : **2×7=14**

(i) Let u and v be distinct vertices of a tree T. Prove that, there is a unique path P between u and v in T.

(ii) Let G be an acyclic graph with n vertices and k components. Prove that, G has $n - k$ edges.